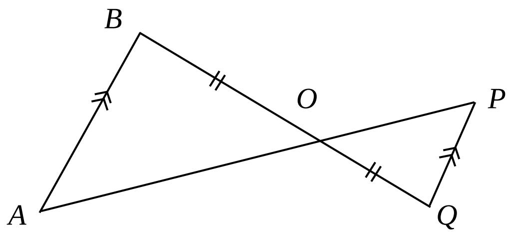
Geometric Proof Assignment – Solutions

Complete Chapter 11 of *Proof: Interesting activities in conjecture and mathematical proof* before attempting this assignment.

1. Triangle *OAB* and triangle *OPQ* are shown on the diagram below. The diagram includes symbols indicating that two lines are parallel and that two line segments are of the same length. The diagram is not drawn to scale.



Required to prove:
$\Delta OAB \cong \Delta OPQ$

Diagram is not to scale.

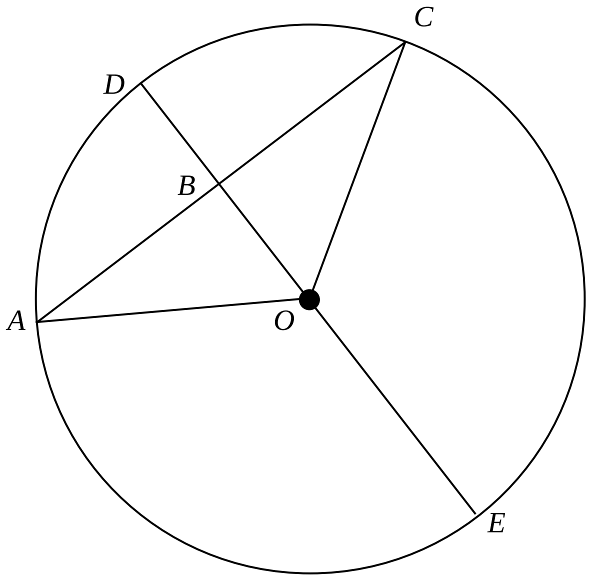
The Explanation column has been left blank in the "two column" proof given below. Complete it from the List of Possible Explanations given below (just fill in the letter of the best Explanation).

Statement	**Explanation**
$\angle BOA \cong \angle QOP$	**A**
$\angle ABO \cong \angle PQO$	**D**
$\overline{OB} \cong \overline{OQ}$	**B**
$\Delta OAB \cong \Delta OPQ$	**M**

List of Some Possible Explanations. (*May be used once, more than once, or not at all.*)
A. Vertically opposite angles are equal.
B. A fact given in the statement of the situation.
C. Parallel lines are the same length.
D. Alternate angles on parallel lines.
E. Programmatic specificity.
F. Pythagoras' Theorem.
G. Corresponding angles on similar shapes are equal.
H. Corresponding line segments on congruent shapes are equal.
I. Lightning never strikes twice in the same place.
J. Symmetry.
K. SSS
L. RHS
M. ASA
N. SAS

2. Required to prove: A diameter perpendicular to a chord bisects the chord.

(a) The circle given below has centre *O* and radius *r*. On the circle draw and label a chord AC, and the diameter DE which is perpendicular to AC and intersects it at B. Also draw and label radius OA and radius OC.

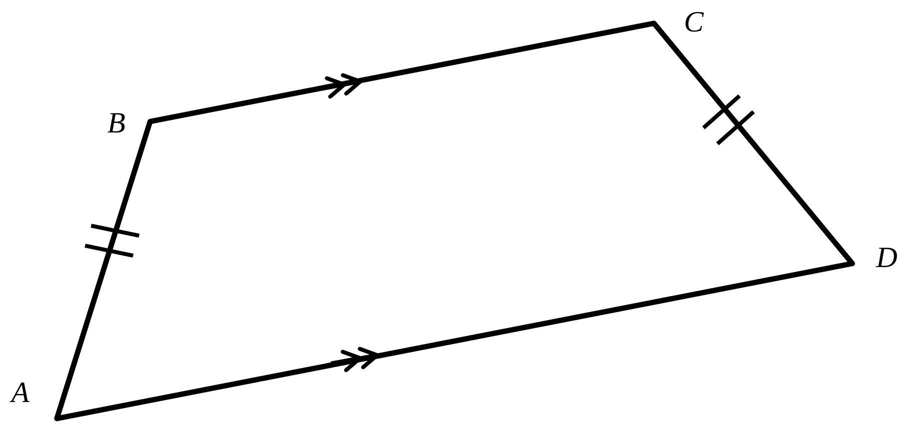


Required to prove: A diameter perpendicular to a chord bisects the chord.

Statement	**Explanation**
$\angle ABO = 90°$	Given. (Diameter is ⊥ to chord.)
$\overline{AO} \cong \overline{CO}$	Both are radii.
$\Delta AOB \cong \Delta COB$	RHS
$\overline{AB} \cong \overline{BC}$	Corresponding line segments on congruent shapes are congruent.

3. An isosceles trapezium has one pair of parallel sides and one pair of congruent sides. It has a single axis of symmetry. Prove that opposite angles are supplementary.

(a) Draw a suitable diagram and use conventional mathematical notation to indicate the given facts.



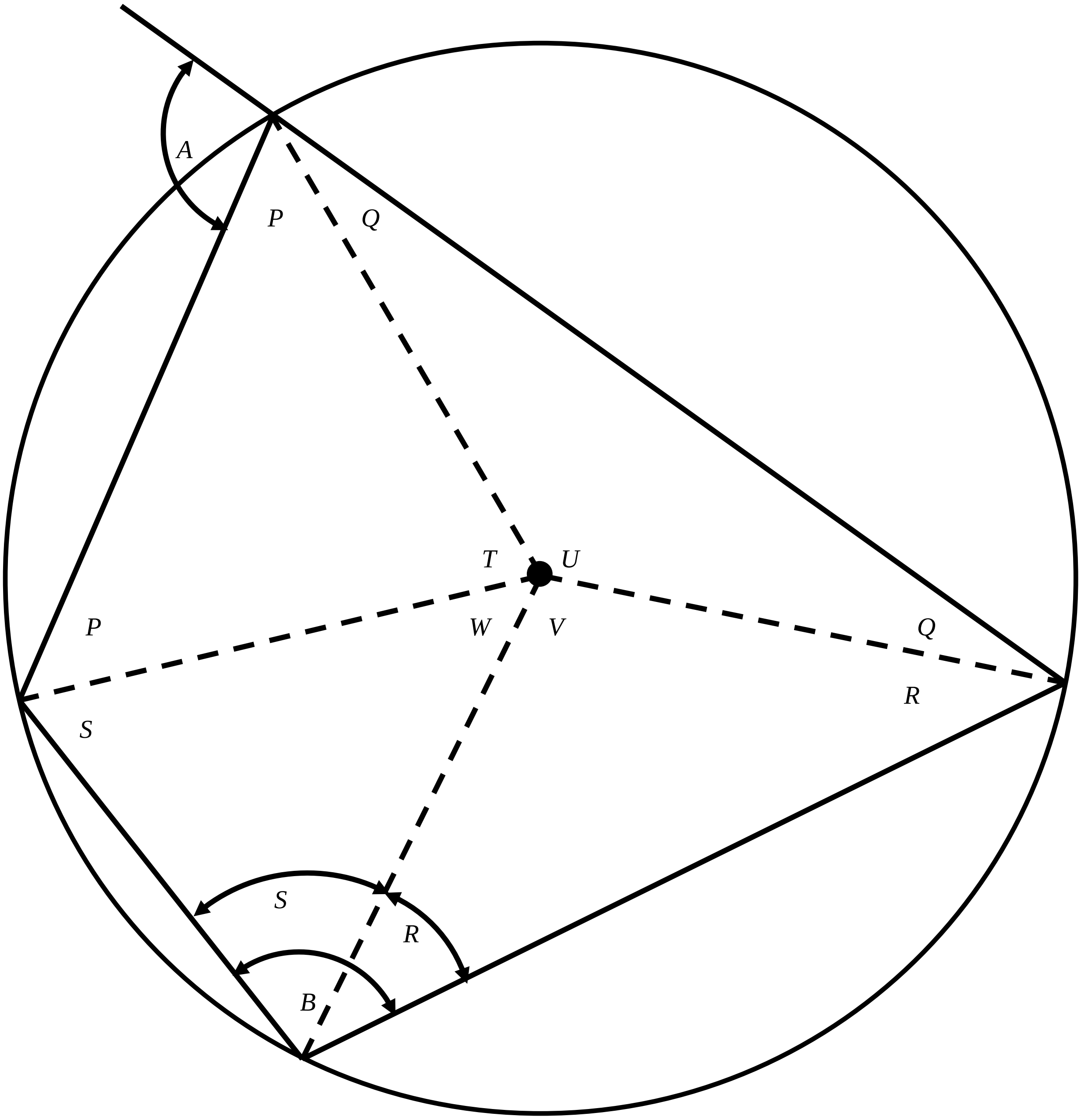
(b) Complete a proof in the table below. There may be more rows than needed. The "Possible Explanations" from Question 1 may be used.

Required to prove: Opposite angles in an isosceles trapezium are supplementary.

Statement	**Explanation**
$\angle BAD \cong \angle CDA$	Symmetry, as stated.
$\angle CBA = 180 - \angle BAD$	Co-interior angles on parallel lines are supplementary.
$\angle CBA = 180 - \angle CDA$	From first statement above.
$\angle CBA + \angle CDA = 180$	As required.

4. Prove that the exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

This problem involves a circle, therefore the diagram should feature a circle. The special property of a circle is that all points on the circumference are equally distant from the centre, therefore it is usually helpful to draw all relevant radii. Such radii often are the equal sides of isosceles triangles.



The letters in the diagram above give the sizes of the angles.

Required to prove: The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

$L.H.S. \;=\; A \;=\; 180 - (P+Q) \;=\; 180 - \left(\frac{180-T}{2} + \frac{180-U}{2}\right) \;=\; \frac{T+U}{2} \;=\; 180 \;-\; \frac{W+V}{2}$

$\;=\; 180 \;-\; \frac{1}{2}(180 - 2S + 180 - 2R) \;=\; S + R \;=\; B \;=\; R.H.S.$

(Other solution formats exist.)

Proof: Interesting activities in conjecture and mathematical proof is available through *www.pbperth.com*

5.

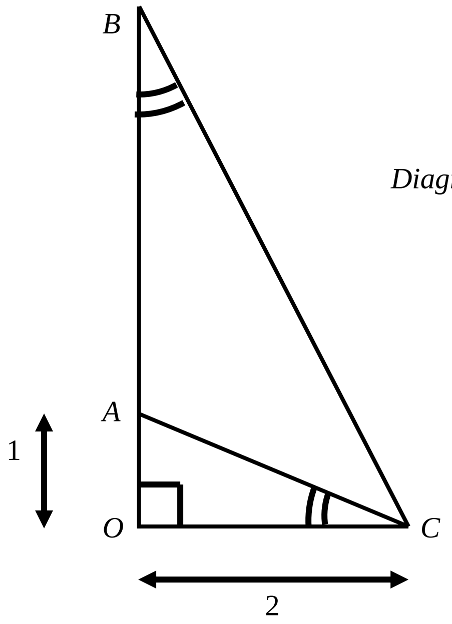


Diagram is not drawn to scale.

Given $\overline{OB} \perp \overline{OC}$ and $\angle OBC \cong \angle OCA$ and $\overline{OA} : \overline{OC} = 1 : 2$

(a) Use conventional mathematical symbols to mark the given information on the diagram.

(b) Prove that $\overline{OA} : \overline{AB} = 1 : 3$

Statement	**Explanation**
$\Delta OBC \sim \Delta OCA$	RHS
$\overline{OA} = 1$	Basis of given ratio.
$\overline{OA} : \overline{OC} = \overline{OC} : \overline{OB}$	Corresponding sides on similar shapes.
$\overline{OB} = 4$	$\frac{\overline{OB}}{\overline{OC}} = \frac{\overline{OC}}{\overline{OA}} \therefore \overline{OB} = \overline{OC} \times \frac{\overline{OC}}{\overline{OA}} = 2 \times \frac{2}{1}$
$\overline{AB} = 3$	$\overline{AB} = \overline{OB} - \overline{OA} = 4 - 1$
$\overline{OA} : \overline{AB} = 1 : 3$	As required.